(1.1-6) Interpret the following matrices as augmented matrices of systems of equations. Write down each system of equations.  

- (c) \[
\begin{bmatrix}
1 & 9 & -3 \\
5 & 0 & 2 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

- (g) \[
\begin{bmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 8 \\
0 & 0 & 1 & 4
\end{bmatrix}
\]

(1.1-7) For the following augmented matrices of systems of equations, find out its solutions.

- (a) \[
\begin{bmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 8 \\
0 & 0 & 0 & 4
\end{bmatrix}
\]
- (b) \[
\begin{bmatrix}
1 & 0 & 1 & 3 \\
0 & 1 & 0 & 8 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
- (c) \[
\begin{bmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 8
\end{bmatrix}
\]

(1.2-2) Consider the system of equations

\[
\begin{align*}
2x_1 & - 6x_2 - 14x_3 = 38 \\
-3x_1 + 7x_2 + 15x_3 & = -37 \\
2x_1 + 2x_2 & + 4x_4 = 0 \\
-2x_1 - 4x_2 + 3x_3 - 2x_4 & = 0
\end{align*}
\]

For what value of \(a\) does the system have one solution, no solution, infinitely many solutions. (第 17 張 投影片)

(1.2-7) Solve (if possible) each the following systems of equations using the method of Gauss-Jordan elimination.

- (b) \[
\begin{align*}
x + 2y & - 3z = 4 \\
3x - y + 5z & = 2 \\
4x + y + (a^2 - 14)z & = a + 2
\end{align*}
\]

(1.3-6) (必須分別檢查 vector addition 和 scalar multiplication) Consider the set of vectors of the following form. Determine which of the sets are subspaces of \(\mathbb{R}^3\).

- (b) \((a, 4a, -3a)\)  
- (d) \((a, 2a + 3b + 6)\)

(1.4-15) Determine a basis for each of the following subspaces of \(\mathbb{R}^3\). Give the dimension of each subspace.

- (b) the set of vectors of the form \((a, a, 2a)\)
- (d) the set of vectors of the form \((a, 2b, a + 3b)\)

(1.5-13) Determine the cosine of the angle between the following pairs of vectors.

- (c) \((2, -1, 0), (5, 3, 1)\)
- (e) \((1, 2, -1, 3, 1), (2, 0, 1, 0, 4)\)

(1.5-22) Let \(W\) be the subspace of vectors of \(\mathbb{R}^3\) that are orthogonal to \(v = (1, -2, 5)\). Find a basis for \(W\). What is the dimension of this subspace? Give a geometrical description of the subspace.
(1.6-5) Determine the equations of the polynomials of degree two whose graphs pass through the given points \((-1, -1), (0, 1),\) and \((1, -3)\). What is the value of \(y\) when \(x = 3\)?

(2.1-25) State (with a brief explanation) whether the following statements as true or false for matrices \(A, B,\) and \(C\).

(b) If the product \(AB\) and \(BC\) exist, then \(AC\) exists.  

(d) If \(A\) is a column matrix and \(B\) a row matrix, both with the same number of elements, then \(AB\) is a square matrix.

(2.2-11) Determine the number of multiplications needed to compute the products \((AB)C\) and \(A(BC)\) when \(A, B,\) and \(C\) are the following sizes:

(b) \(A 3 \times 7,\ B 7 \times 5,\ C 5 \times 2\)

(2.2-29) Prove that if a matrix \(A\) commutes with a diagonal matrix that has no two diagonal elements the same, then \(A\) is a diagonal matrix.  (Hint: \(A\) commutes with \(B\) if \(A\ B = B\ A\).)

(2.2-32) A square matrix is said to be idempotent if \(A^2 = A\). Determine \(a, c,\) and \(d\) such that \[
\begin{bmatrix}
    a & 0 \\
    c & d
\end{bmatrix}
\]
is idempotent.

(2.3-1) Prove that \((A\ B)^t = B^t A^t\) for 2 by 2 matrices \(A\) and \(B\).  (Note that this equality holds for any size of matrices.)

(2.3-11) A matrix \(A\) is said to be **antisymmetric** if \(A = -A^t\).

(b) Prove that an antisymmetric matrix is a square matrix having diagonal elements zero.

(d) Prove that the scalar multiple of an antisymmetric matrix is antisymmetric.

(2.3-12) Assume that \(A\) and \(B\) are 3 by 3 matrices. Prove that \(\text{tr} (AB) = \text{tr} (BA)\). (Note that this equality holds for any size of matrices.)

(2.3-20) Prove that \(A\ B = 0_n\) for all \(n \times n\) matrices \(B\) if and only if \(A = 0_n\).

(2.4-4) Determine the inverse of each of the following 3 \(\times\) 3 matrices, if it exist, using the method of Gauss-Jordan elimination.

(b) \[
\begin{bmatrix}
    2 & 0 & 4 \\
    -1 & 3 & 1 \\
    0 & 1 & 2
\end{bmatrix}
\]

(d) \[
\begin{bmatrix}
    1 & 2 & -1 \\
    3 & -1 & 0 \\
    2 & -3 & 1
\end{bmatrix}
\]

(2.4-12) Prove the following properties of matrix inverse.

(b) \((A^n)^{-1} = (A^{-1})^n\)  

(c) \((A^t)^{-1} = (A^{-1})^t\)

(2.4-22) Is the result \((A + B)^{-1} \neq A^{-1} + B^{-1}\) true in general? Prove it if it is true or construct a counterexample if it is wrong.

(2.4-26) Prove that a diagonal matrix is invertible if and only if all its diagonal elements are nonzero. Can you find a quick way for determining the inverse of an invertible diagonal matrix?
(2.4-31) Let $T_1$ and $T_2$ be the following row operations. $T_1$: multiply row 1 by $-3$. $T_2$: add 3 times row 2 to row 1. Find the elementary $3 \times 3$ matrices of $T_1$, $T_2$.

(2.5-10) (b) Let $T_1(x) = A_1x$ and $T_2(x) = A_2x$ be defined by the following matrices:

$$A_1 = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & -1 \end{bmatrix} , A_2 = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}.$$ Let $T = T_2 \circ T_1$. Find the matrix that defines $T$ and use it to determine the image of the vector $x = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ under $T$.

(2.5-15) Let $A$ be the rotation matrix for $\pi/4$. Show that $A^8 = I$, the identity $2 \times 2$ matrix. Give a geometrical reason for expecting this result.

(2.5-19) Show that the following matrix $A$ is orthogonal. Show that the transformation defined by $A$ preserves the norms of the vectors $u$ and $v$, preserves the angle between these vectors, and also preserves the distance between the points defined by the vectors.

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} , u = \begin{bmatrix} 3 \\ 3 \end{bmatrix} , v = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

(2.5-31) (c) Construct single $2 \times 2$ matrix that defines the following transformations on $\mathbb{R}^2$.

Find the image of the point $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ under this transformation.

A dilation of factor 2, then a shear of factor 3 in the $x$ direction, then a rotation through $\pi/2$ counterclockwise. (Hint: A shear of factor $c$ in the $x$-direction is

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + cy \\ y \end{bmatrix}$$

(2.6-5) (必須分別檢查 2 個條件) Prove that $T: \mathbb{R}^2 \to \mathbb{R}$ defined by $T(x, y) = x + a$, where $a$ is a nonzero scalar, is not linear.

(2.6-7) (必須分別檢查 2 個條件) Determine whether the given transformation is linear.

$T(x, y) = x - y$ of $\mathbb{R}^2 \to \mathbb{R}$

(2.6-17) Let $T_1(x, y) = (3x + y, 4y)$ and $T_2(x, y) = (2y, x - y)$. Let $T = T_2 \circ T_1$.

(1) Find an equation for $T$ and use it to determine the image of $(1, 2)$ under $T$.

(2) Find the standard matrices for these 3 transformations.

(2.8-2) Prove that the product of two $2 \times 2$ stochastic matrices is a stochastic matrix. (Note that this equality holds for any size of matrices.) (Hint: $A = \begin{bmatrix} x & y \\ 1-x & 1-y \end{bmatrix} , B = \begin{bmatrix} u & w \\ 1-u & 1-w \end{bmatrix}$ are stochastic matrices. Prove that $AB$ is stochastic.)

(2.8-3) Find the 1-step transition matrix for the following transition diagram of a certain Markov chain.
(2.8-12) The following matrix gives occupational transition probabilities.

\[
\begin{array}{cc}
\text{nonfarming} & \text{farming} \\
1 & 0.4 \\
0 & 0.6
\end{array}
\]

(initial generation)

\[
\begin{array}{cc}
\text{nonfarming} & \text{farming} \\
1 & 0.4 \\
0 & 0.6
\end{array}
\]  

(2.9-3) (b) (1) Sketch the digraphs with the adjacency matrix

\[
\begin{bmatrix}
0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{bmatrix}.
\]

(2) What is the total number of 2-paths?

(2.9-8) (b) The matrix \(A\) is the adjacency matrix for a communication network.

(1) Sketch the networks.  
(2) Interpret the elements \(A^2(2, 4), A^2(3, 2), A^2(4, 2), A^3(1, 3), A^3(3, 2),\) and \(A^3(3, 4),\)

\[
A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}, \quad A^2 = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}, \quad A^3 = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

(3.1-13) Solve the equation \[
\begin{bmatrix} 2x \\ x - 1 \\ x + 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \
\] for \(x\).

(3.1-18) 考慮 \[
A = \begin{bmatrix}
3 & 0 & 1 & 0 \\
0 & -2 & 1 & 0 \\
-2 & 1 & 0 & 2 \\
0 & -1 & 1 & 1
\end{bmatrix}
\]

(1) 使用 cofactor expansion，針對第三列 (row)，求 \(|A|\)
(2) 使用 cofactor expansion，針對第四行 (column)，求 $|A|$.

(3) 左對角減右對角

(3.2-10) If $A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ and $|A| = 3$, compute the determinants of the following matrices.

(b) $\begin{vmatrix} d & a & g \\ e & b & h \\ f & c & i \end{vmatrix}$

(d) $\begin{vmatrix} d & e & f \\ a & b & c \\ 2g & 2h & 2i \end{vmatrix}$

(3.2-19) Let A and B be square matrices of the same size. Is the result $|A + B| = |A| + |B|$ true in general? Prove it if it is true or construct a counterexample if it is wrong.

(3.2-20) Find det (A) if $A$ is an $n \times n$ matrix defined by

$A = I_n + a \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+a & a & \cdots & a \\ a & 1+a & a & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ a & a & a & \cdots & 1+a \end{bmatrix}$

(3.2-21) (b) Evaluate the determinant $\begin{vmatrix} 1 & 0 & -2 & 1 \\ 2 & 1 & 0 & 2 \\ -1 & 1 & -2 & 1 \\ 3 & 1 & -1 & 0 \end{vmatrix}$ using elementary row operations.

(3.2-23) 當我們定義對角矩陣為此形式 $A_n = \begin{bmatrix} 0 & \cdots & \cdots & 0 & a_1 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & a_{n-1} & 0 \\ a_n & 0 & \cdots & \cdots & 0 \end{bmatrix}$，Theorem 3.4-1

將不成立：

(1) 求 $|A_2| = \begin{vmatrix} 0 & a_1 \\ a_2 & 0 \end{vmatrix}$

(2) 求 $|A_3|$

(3) 求 $|A_4|$

(3.3-10) (b) Solve the following systems of equations using Cramer’s rule.

$x_1 + 2x_2 + x_3 = 9$
$x_1 + 3x_2 - x_3 = 4$
$x_1 + 4x_2 - x_3 = 7$
(3.3-16) Determine the values of $\lambda$ for which the following system of equations has nontrivial solutions. Find the solutions for each value of $\lambda$.

\[
\begin{align*}
(\lambda + 4)x_1 + (\lambda - 2)x_2 &= 0 \\
4x_1 + (\lambda - 3)x_2 &= 0
\end{align*}
\]

(3.3-23) Prove that if $A$ and $B$ are square matrices of the same size, with $AB$ being invertible, then $A$ and $B$ are invertible. Is the converse true?

(3.3-28) State (with a brief explanation) whether the following statements are true or false for square matrices $A$, $B$, and $C$ of the same size.

(b) If a diagonal matrix $A$ is singular, then at least one diagonal element is zero.

(d) If $|A - B| = 0$, then $|A| = |B|$.

(3.3-29) 当 $n = 4$，证明 Vandermonde Matrix 的 determinant 公式

(7.2-7) (b) Let $A = LU$. The following sequence $R_2 + 5R_1$, $R_3 - 2R_1$, $R_3 + 7R_2$ of transformations are used to arrive at $U$. Find $L$.

(7.2-10) Solve the systems using the method of $LU$ decomposition.

\[
\begin{align*}
2x_1 + x_2 &= 9 \\
6x_1 + 4x_2 - x_3 &= 25 \\
2x_1 + 4x_3 &= 20
\end{align*}
\]

(7.2-13) Solve the systems using the method of $LU$ decomposition.

\[
\begin{align*}
4x_1 + x_2 + 2x_3 &= 18 \\
-12x_1 - 4x_2 - 3x_3 &= -56 \\
-5x_2 + 16x_3 &= -10
\end{align*}
\]

(7.2-25) Consider the matrix

\[
E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}
\]

that defines the interchange of row 2 and 3 of a $3 \times 3$ matrix. Show that $E$ has no LU decomposition. This example demonstrates that not every matrix can be written in the form $LU$. (Hint: Assume it exists, but you will reach a contradiction.)

(4.1-8) Is the set of $3 \times 3$ symmetric matrices a vector space?

(4.1-9) Is the set $2 \times 2$ stochastic matrices a vector space? (See Section 2.8 for the definition of stochastic matrix.)

(4.1-13) Consider the set of polynomial functions of degree 2. Is this set a vector space?

(4.1-19) (必须分别检查 vector addition 和 scalar multiplication) Determine which of the following subsets of $M_{22}$ form subspaces.

(b) The subset consisting of matrices the sum of whose elements is 6. (For
example, \[
\begin{bmatrix}
2 & -1 \\
0 & 5
\end{bmatrix}
\] would be such a matrix.

(d) The subset of matrices of the form \[
\begin{bmatrix}
a & a+2 \\
b & c
\end{bmatrix}
\]

(4.2-3) In the following sets of vectors, determine whether the first vector is a linear combination of the other vectors.
   (b) \((-2, 11, 7); (1, -1, 0), (2, 1, 4), (-2, 4, 1)\)
   (c) \((2, 7, 13); (1, 2, 3), (-1, 2, 4), (1, 6, 10)\)

(4.2-24) (a) Is the function \(f(x) = x + 5\) in the vector space generated by \(g(x) = x + 1\) and \(h(x) = x + 3\) ?
   (b) Is the function \(f(x) = 3x^2 + 5x + 1\) in the vector space generated by \(g(x) = 2x^2 + 3\) and \(h(x) = x^2 + 3x - 1\)?
   (c) Give three other functions in the vector space generated by \(g(x) = 2x^2 + 3\) and \(h(x) = x^2 + 3x - 1\).

(4.3-10) (b) Determine whether or not the following set
\[
\begin{bmatrix}
1 & 2 \\
3 & 1 \\
2 & 4
\end{bmatrix}
\]
are linearly dependent in \(M_{22}\).

(4.3-11) (b) Determine whether or not the following sets of functions \(f(x) = x^2 + 3\), \(g(x) = x + 1\), \(h(x) = 2x^2 - 3x + 3\) are linearly dependent in \(P_2\).

(4.4-20) (b) Is the following set \(f(x) = x^2 + x - 3\), \(g(x) = x^2 - x + 1\), \(h(x) = x^2 + x - 1\) a basis for the vector space of \(P_2\)?

(4.4-22) Let \(\{v_1, v_2, v_3\}\) be a basis for a vector space \(V\). Show that the set of vectors
\[
\{u_1, u_2, u_3\}, \text{ where } u_1 = v_1, \text{ and } u_2 = v_1 + v_2, \text{ } u_3 = v_1 + v_2 + v_3, \text{ is also a basis for } V.
\]

(4.5-5) (c) Find the reduced echelon form for the matrix
\[
\begin{bmatrix}
1 & 1 & 0 & -1 \\
2 & 1 & 0 & 0 \\
3 & 2 & 0 & -1 \\
-1 & 0 & 1 & 1
\end{bmatrix}
\].
Use the reduced echelon form to determine a basis for the row space and rank of the matrix.

(4.5-9) Find bases for both the row and column spaces of the following matrix \(A\). Show that the dimensions of both the row space and the column space are the same. For each row, please represent it as a linear combination of the vectors of the row
For each column, please represent it as a linear combination of the vectors of the column space.

\[
A = \begin{bmatrix}
1 & 3 & 2 \\
1 & 4 & 1 \\
2 & 5 & 5
\end{bmatrix}
\]

(4.6-1) Determine the projection of \((x, y)\) onto the line \(y = m x\). (Hint: The vector representing the line is \((1, m)\).)

(4.6-2) Find the QR-decomposition of \(A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\).

(4.6-9) Let \(A\) and \(B\) be orthogonal matrices of the same size. Prove that the product matrix \(AB\) is also an orthogonal matrix.

(4.6-16) (b) The vectors \((3, 2, 0)\), \((1, 5, -1)\), \((5, -1, 2)\) form a basis for \(R^3\). Use these vectors in the Gram-Schmidt process to construct an orthonormal basis for \(R^3\).

(4.6-27) Find the distance of the point \(x = (2, 4, -1)\) of \(R^3\) from the subspace \(W\) consisting of all vectors of the form \((a, -2a, b)\).

(6.4-21) Determine the least-squares parabola for the data points \((1, 5)\), \((2, 2)\), \((3, 3)\), \((4, 8)\).

(6.4-27) A state highway company has arrived at the following statistic on age of driver and traffic fatalities. Determine the least-squares line and use it to predict the number of driver fatalities at age 22.

<table>
<thead>
<tr>
<th>Age</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of fatalities</td>
<td>101</td>
<td>115</td>
<td>92</td>
<td>64</td>
<td>60</td>
<td>50</td>
<td>49</td>
</tr>
</tbody>
</table>

(6.4-35) Consider the subspace \(W\) generated by the vectors \((1, 1, 1)\) and \((2, 1, 0)\). Find the projection matrix for \(W\) and use it to find projection of the vector \((3, 0, 1)\) onto \(W\).

Show that the projection of \((6, 3, 0)\) onto \(W\) is, in fact, \((6, 3, 0)\). What does this mean?

(4.7-5) Determine the kernel and range of the following transformation \(T(x, y) = (x, 2x, 3x)\) of \(R^2 \rightarrow R^3\). Show that dim ker(T) + dim range(T) = dim domain(T).

(4.7-19) Prove that \(T: P_3 \rightarrow P_2\), defined as follows, is linear. Find the kernel and range of \(T\).

\[T(a_3 x^3 + a_2 x^2 + a_1 x + a_0) = a_3 x^2 - a_0\]

(4.7-29) Let \(T: M_{22} \rightarrow M_{22}\) be the linear operator defined by \(T(X) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} X\), where \(M_{22}\) represents the vector space of all \(2 \times 2\) matrices.

Find the rank and nullity of \(T\).

(4.9-14) Solve the differential equation \(d^2y/dx^2 + 8 dy/dx = 3 e^{2x}\)

(4.9-15) Given \(x1 - 2x2 + 3x3 = 1\)
Find an arbitrary solution \( x_g \), the kernel \( x_k \), and a particular solution \( x_p \).

Verify that \( A x_k = 0 \) and \( A x_p = y \).

(3.4-11) Determine the characteristic polynomials, eigenvalues, and corresponding eigenspaces of

\[
\begin{bmatrix}
1 & 0 & 0 \\
-2 & 1 & 2 \\
-2 & 0 & 3
\end{bmatrix}
\]

(3.4-34) (Cayley-Hamilton theorem) If the characteristic equation of a square matrix \( A \) is

\[\lambda^n + c_{n-1}\lambda^{n-1} + \ldots + c_0 = 0, \] then \( A^n + c_{n-1}A^{n-1} + \ldots + c_0 I = 0. \) (1) Show that matrix \( B = \begin{bmatrix} -1 & 5 \\ -10 & 14 \end{bmatrix} \) satisfies its characteristic equation. (2) Compute \( B^{100} \)

(5.5-1) Consider each of the following matrices. Determine which one is positive definite, positive semidefinite, negative definite, negative semidefinite, or indefinite.

\[
(a) \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}, \quad (b) \begin{bmatrix} 2 & 4 \\ 4 & 1 \end{bmatrix}, \quad (c) \begin{bmatrix} -2 & 0 & 1 \\ 0 & -3 & 0 \\ 1 & 0 & -2 \end{bmatrix}, \quad (d) \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}, \quad (e) \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & 2 & -5 \end{bmatrix}
\]

(5.5-2) For any matrix \( A \in \mathbb{R}^{m \times n} \), prove that \( A' A \) and \( A A' \) are positive semi-definite.

(5.5-3) Given a positive semi-definite and an invertible matrix \( B \), prove that it is positive definite.

(5.5-4) Express the following quadratic forms in the matrix notation \( x' A x \), where \( A \) is a symmetric matrix.

(a) \( 9x_1^2 - x_2^2 + 4x_3^2 + 6x_1 x_2 - 8x_1 x_3 + x_2 x_3 \)

(b) \( x_1 x_2 + x_1 x_3 + x_2 x_3 \)

(5.5-5) Find the critical point(s) of the function \( f(x, y) = xy + \frac{4}{x} + \frac{2}{y} \). Then use Sylvester’s Criterion to classify the nature of each point, if possible. Finally, determine the relative extrema of the function.

(5.3-8) Orthogonally diagonalize the symmetric matrix

\[
\begin{bmatrix}
1 & 2 & -2 \\
2 & 4 & -4 \\
-2 & -4 & 4
\end{bmatrix}
\]

by the similarity transformation.

(5.3-11) Prove that if \( A \) and \( B \) are similar matrices, then \( A' \) and \( B' \) are similar.

(5.3-18) Show that if \( A \) and \( B \) are orthogonally similar and \( B \) and \( C \) are orthogonally similar, then \( A \) and \( C \) are orthogonally similar.
(5.3-20) If \( A^3 = \begin{bmatrix} 83 & 84 \\ 42 & 41 \end{bmatrix} \), compute \( A^5 \). (Hint: Diagonalize \( A^3 \). Use \( A^3 \) to get \( A \), then finally compute \( A^5 \).)

(3.5-8) A market research group has been studying the buying patterns for three competing product I, II, and III. The result of the analysis are described by the following matrix.

\[
P = \begin{bmatrix} 80\% & 20\% & 5\% \\ 5\% & 75\% & 5\% \\ 15\% & 5\% & 90\% \end{bmatrix}
\]

Column 1 implies that of those people currently using product I, 80% plan to continue using it, while 5% plan to switch to product II and 15% to product III. Column 2 and 3 are to be interpreted similarly. Use the matrix P to construct a Markov chain that describes buying trends. If the current buying patterns continue, determine the likely eventual distribution of sales, in terms of percentages.

(3.5-9) Consider a small web consisting of four pages A, B, C, and D.

(a) What is the transition matrix M?
(b) Is the matrix M regular? If yes, please explain. (Hint: You could use Excel or any program to compute it. For Excel, the command is MMULT.)
(c) Solve directly by using the method of Gauss-Jordan elimination to get the numbers of the pagerank vector. Which web page is more important?
(d) Assume the initial page rank vector is \( PR(0) = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \). Please compute the first 10 iterations of \( PR(k+1) = M \cdot PR(k) \). You will see how fast it converges!
   (Hint: You could use Excel or any program. One “for” loop and matrix data structure are enough.)

(5.4-3) (b) Solve the following difference equation \( a_n \) and use the solutions to determine the term \( a_{1000} \).
\[ a_n = 2a_{n-1} + 3a_{n-2}, \ a_1 = 1, \ a_2 = 3. \] Find \( a_n \) and \( a_{1000} \).

\[(5.4-4) \text{ dy } \frac{dt}{t} = \begin{bmatrix} 5 & -4 \\ 2 & -1 \end{bmatrix} y(t), \text{ 其中 } y(0) = [1 \ -1]^T \]

\[(5.4-5) \text{ Solve } a_n + 4a_{n-1} + 3a_{n-2} = n \]

\[(5.4-7) \text{ (來源: Dan Brown 著、尤傳莉譯, 達文西密碼, 時報出版, Section 20) 找 5 位修這門課的同學, 5 位沒有修這門課的同學, 列出其班級、學號、姓名, 計算肩膀到指尖的長度除以手肘到指尖的長度, 然後求 10 位同學的平均值 (算到小數點後面三位) } \]